“Newton Versus Descartes on the Exactness of Mathematics”

David Bakker — MC-PMP 2025, Brussels — Handout

A) Preface to the *Principia* (Published in 1687)

“*…* But since those who practice [practical mechanics] do not generally work with a high degree of exactness, the whole subject of *mechanics* is distinguished from *geometry* by the attribution of exactness to *geometry* and of anything less than exactness to *mechanics.* Yet the errors do not come from the art but from those who practice the art. Anyone who works with less exactness is a more imperfect mechanic, and if anyone could work with the greatest exactness, he would be the most perfect mechanic of all. For the description of straight lines and circles, which is the foundation of *geometry*, appertains to *mechanics*. *Geometry* does not teach how to describe these straight lines and circles, but postulates such a description. For *geometry* postulates that a beginner has learned to describe lines and circles exactly before he approaches the threshold of *geometry,* and then it teaches how problems are solved by these operations. To describe straight lines and to describe circles are problems, but not problems in geometry. Geometry postulates the solution of these problems from *mechanics* and teaches the use of the problems thus solved. And *geometry* can boast that with so few principles obtained from other fields, it can do so much. Therefore *geometry* is founded on mechanical practice and is nothing other than that part of *universal mechanics* which reduces the art of measuring to exact propositions and demonstrations.” [pp.27-28]

Source:

Newton, Isaac. 2016. *The Principia: The Authoritative Translation: Mathematical Principles of Natural Philosophy.* Tr. I. Bernard Cohen and Anne Whitman. University of California Press.

B) “Geometry”: the first Book (Unpublished manuscript, ca. 1690s)

“Geometry neither teaches how to describe a plane nor postulates its description, though this is its whole foundation. To be sure, the planes of fields are not formed by the practitioner but merely measured. Geometry does not teach how to describe a straight line and a circle but postulates them; in other words, it postulates that the practitioner has learnt these operations before he attains the threshold of geometry. Once, however, these are previously understood and granted, it teaches all the other operations of mensuration under the head of ‘problems’, and likewise under the head of ‘theorems’ compares quantities with one another in order from the measured and the found to derive things which have not been measured.

… Geometry does not posit modes of description: we are free to describe [geometrical figures] by moving rulers around, using optical rays, taut threads, compasses, the angle given in a circumference, points separately ascertained, the unfettered motion of a careful hand, or finally any mechanical means whatever. Geometry makes the unique demand that they be described exactly. It has now, however, come to be usual to regard as geometrical everything which is exact, and as mechanical all that proves not to be of the kind, as though nothing could possibly be mechanical and at the same time exact. But this common belief is a stupid one, and has its origin in nothing else than that geometry postulates an exact mechanical practice in the description of a straight line and a circle, and moreover is exact in all its operations, while mechanics as it is commonly exercised is imperfect and without exact laws. It is from the ignorance and imperfection of mechanicians that the common opinion defines mechanics. On this reasoning a thing would be the more mechanical the more imperfect it was. Posit a mechanical thing to be perfect and you will correct the error. For assuredly the more mechanical—that is, skillfully wrought—a thing is, the more exact it is, and he the more perfect mechanic who works the more perfectly and exactly, while he alone is perfect who works exactly.” [pp.287-289]

“Geometry and mechanics are distinguished not inasmuch as they are more and less exact, but in the use and end of their disciplines. The purpose of mechanics is to form and move magnitudes in appointed figures and motions: that of geometry is neither to form nor move magnitudes, but merely to measure them. Geometry forms nothing except modes of measuring. It postulates a technician who knows how to form straight lines and circles, and teaches him how through their formation appointed magnitudes are to be measured.” [p.291]

“Geometry postulates the faculty not of pondering but of performing work: it does not postulate the subject on which it shall act but merely defines it. It was, of course, devised not to establish magnitudes to be measured, but to measure them once established.” [p.297]

“Geometry is required for Mechanics, and, in turn, Geometry rests upon the operations of Mechanics.” [p.338]

Source:

Newton, Isaac. 1976. *The Mathematical Papers of Isaac Newton. Vol. 7: 1691–1695*. Ed. D. T. Whiteside. Cambridge: Cambridge University Press.

C) Miscellaneous Mathematical Reflections (Unpublished manuscript, ca. 1705-6)

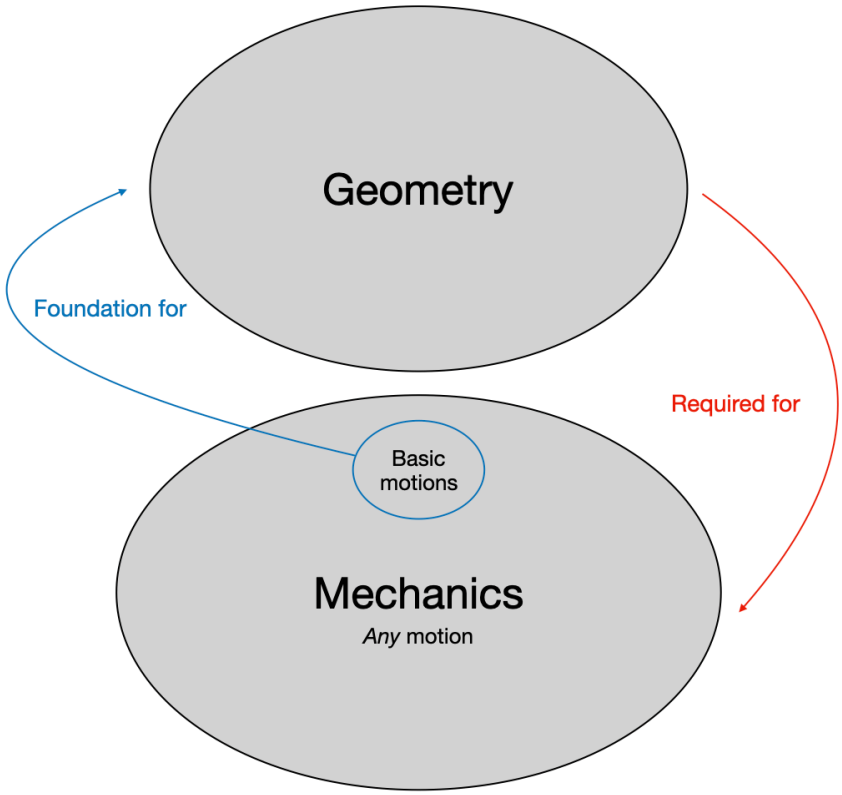
"All things are numbered, magnitudes measured, and bodies moved, and out of these operations there arise arithmetic, geometry and mechanics. These are sciences inasmuch as they teach the truth by means of definitions, axioms and theorems, but arts (skills) insofar as they deliver and exhibit its practice by means of postulates and constructions of problems.

Mechanics holds a place among the mathematical sciences through its axioms and demonstrations of a very great many truths, but not so among the mathematical arts. Its practice is not founded on postulates but is purely manual. Geometry and arithmetic are both mathematical arts and sciences.

Geometry, inasmuch as it is a science, considers all magnitudes and figures which can be defined, and discloses their properties and proportions by means of axioms and theorems; insofar as it is an art, it postulates that its practitioner shall carefully have learnt the prime and easiest operations of all, and those most necessary, expedient and useful in the technique of measurement, and by means of postulates and constructions of problems thereby demonstrated it determines and sets forth measures of all figures and magnitudes which are proposed by the definitions.

Arithmetic, inasmuch as it is a science, discloses the laws and properties of numbers and proportions by means of axioms and theorems: insofar as it is an art, it teaches the operations of adding, subtracting, multiplying, dividing and extracting roots (with, obviously, the necessary postulates understood), and by means of these operations it tracks down and determines all numbers sought and all their proportions, and by expressing magnitudes, motions, forces, times and all other quantities through numbers it considers and determines geometrical, mechanical and all the other problems which can be propounded about the quantities and proportions of things." [p.175]

Source:
Newton, Isaac. 1981. *The Mathematical Papers of Isaac Newton. Vol. 8: 1697–1722.* Ed. D. T. Whiteside. Cambridge: Cambridge University Press.



Newton's perspective of the relationship of mechanics and geometry